Polarization effects on RT simulation of spectral radiance for CLARREO solar benchmark

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Objective: To quantify the polarization error in spectral radiance/reflectance from scalar radiative transfer (SRT) computations, and therefore, the polarization impact on simulation of solar reflectance benchmark for CLARREO.

- What’s the spectral characteristic of this polarization error? Where does the largest error occur?
- How does the polarization error in the SRT spectral radiance/reflectance change with the sun-view geometry?
- How does the surface condition affect the polarization error?
- Under what situations can we neglect the polarization and use the simpler and efficient scalar RT calculation? Or when does the polarization have to be taken into account?
From scalar radiative transfer (SRT) to VRT: Comparison of RT equations

<table>
<thead>
<tr>
<th>Scalar</th>
<th>Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. General RTE</strong></td>
<td><strong>2. Solar source</strong></td>
</tr>
<tr>
<td>$\frac{dI(\tau,\mu,\varphi)}{d\tau} = I(\tau,\mu,\varphi) - \frac{\omega(\tau)}{4\pi} \int_{0}^{2\pi} d\varphi' \times \int_{-1}^{1} p(\tau,\mu,\varphi,\mu',\varphi')I(\tau,\mu',\varphi')d\mu' + S(\tau,\mu,\varphi)$</td>
<td>$\frac{d\tilde{I}(\tau,\mu,\varphi)}{d\tau} = \tilde{I}(\tau,\mu,\varphi) - \frac{\omega(\tau)}{4\pi} \int_{0}^{2\pi} d\varphi' \times \int_{-1}^{1} \hat{P}(\tau,\mu,\varphi,\mu',\varphi')\tilde{I}(\tau,\mu',\varphi')d\mu' + \hat{S}(\tau,\mu,\varphi)$</td>
</tr>
<tr>
<td>$I$ and $S$ are scalar. $P$ is phase function and $P = \tilde{P}_{1,1}$</td>
<td>$\tilde{I} = (I,Q,U,V)^T$</td>
</tr>
<tr>
<td>$\hat{P}(\tau,\mu,\varphi,\mu',\varphi') = T(\pi-\chi_2)\hat{P}(\Theta)T(\chi_1)$</td>
<td></td>
</tr>
<tr>
<td><strong>3. Solar source</strong></td>
<td><strong>In atmosphere</strong></td>
</tr>
<tr>
<td><strong>Atm-surface</strong></td>
<td>$S(\tau,\mu,\varphi) = \frac{\omega(\tau)}{4\pi} F_0 p(\tau,\mu,\varphi,-\mu_0,\varphi_0)\exp(-\tau/\mu_0)$</td>
</tr>
<tr>
<td>In atmosphere</td>
<td>$\hat{S}(\tau,\mu,\varphi) = \frac{\omega(\tau)}{4\pi} \tilde{F}_0 \hat{P}(\tau,\mu,\varphi,-\mu_0,\varphi_0)\exp(-\tau/\mu_0)$</td>
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<tr>
<td><strong>Atm-ocean system</strong></td>
<td>In ocean</td>
</tr>
<tr>
<td>$S(\tau,\mu,\varphi) = \frac{\omega(\tau)}{4\pi} \frac{\mu_0}{\mu_{0m}} F_0 p(\tau,\sum_{m=0}^{\infty} \mu_0,\varphi,-\mu_0,\varphi_0)\exp(-\frac{\tau_{am}}{\mu_0}) \times T(-\mu_0,\varphi_0,-\mu_0,\varphi_0,w,n)\exp(-\frac{\tau-\tau_{am}}{\mu_0})$</td>
<td>$\hat{S}(\tau,\mu,\varphi) = \frac{\omega(\tau)}{4\pi} \frac{\mu_0}{\mu_{0m}} \tilde{F}<em>0 \hat{P}(\tau,\mu,\varphi,-\mu_0,\varphi_0)\exp(-\frac{\tau</em>{am}}{\mu_0}) \times \hat{T}(-\mu_0,\varphi_0,-\mu_0,\varphi_0,w,n)\exp(-\frac{\tau-\tau_{am}}{\mu_0})$</td>
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</table>
\[ I(\tau, \mu, \varphi) = \sum_{m=0}^{2N-1} I^m(\tau, \mu) \cos(m(\varphi - \varphi_0)) \]

4. Fourier space

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<td>( \tilde{I}(\tau, \mu, \varphi) = \frac{1}{2} \sum_{m=0}^{2N-1} (2 - \delta_{0m}) \Phi^m(\varphi - \varphi_0) \tilde{I}^m(\tau, \mu) )</td>
<td>( \Phi^m(\varphi) = \text{diag} { \cos m\varphi, \cos m\varphi, \sin m\varphi, \sin m\varphi } ) (Simplified for illumination by natural light)</td>
</tr>
</tbody>
</table>

5. Phase function /matrix decomposition

\[
p(\mu, \varphi, \mu', \varphi') = \sum_{l=0}^{\infty} (2l + 1) g_l \{ P_l(\mu)P_l(\mu') \\
+ 2 \sum_{m=1}^{l} \frac{(l - m)!}{(l + m)!} P_l^m(\mu)P_l^m(\mu') \cos m(\varphi - \varphi') \}
\]

\[
P_l^m = \hat{P}_l^m[1,1]
\]

\( P_l^m \): Associated Legendre polynomial and is real.

\[
\hat{p}(\mu, \varphi, \mu', \varphi') = \frac{1}{2} \sum_{l=0}^{\infty} (2 - \delta_{0m}) \left[ C^m(\mu, \mu') \cos m(\varphi - \varphi') + S^m(\mu, \mu') \sin m(\varphi - \varphi') \right]
\]

\[
C^m(\mu, \mu') = A^m(\mu, \mu') + DA^m(\mu, \mu')D
\]

\[
S^m(\mu, \mu') = A^m(\mu, \mu') - DA^m(\mu, \mu')
\]

\[
A^m(\mu, \mu') = \sum_{l=0}^{\infty} \hat{P}_l^m(\mu)B_l \hat{P}_l^m(\mu'); D = \text{diag}\{1,1,-1,-1\}
\]

\( \hat{P}_l^m \) is a matrix and contains complex entries.

6. Discrete ordinate equation

\[
\pm \mu_i \frac{dI^m(\tau, \pm \mu_i)}{d\tau} = -I^m(\tau, \pm \mu_i) - \sum_{j=1}^{N} w_j \\
\left[ D^m(\tau, \pm \mu_i, \mu_j)I^m(\tau, \mu_j) + D^m(\tau, \pm \mu_i, -\mu_j)I^m(\tau, -\mu_j) \right] \\
+ S^m(\tau, \pm \mu_i, \mu_0); \quad i = 1, 2, \cdots, N
\]

\[
D^m(\tau, \mu, \mu') = \frac{\omega(\tau)}{2} \sum_{l=0}^{2N-1} (2l + 1) g_l(\tau) \frac{(l - m)!}{(l + m)!} P_l^m(\mu)P_l^m(\mu')
\]

\[
\pm \mu_i \frac{d\tilde{I}^m(\tau, \pm \mu_i)}{d\tau} = -\tilde{I}^m(\tau, \pm \mu_i) - \frac{\omega(\tau)}{2} \sum_{l=0}^{2N-1} \tilde{\hat{P}}_l^m(\pm \mu_i)B_l \\
\sum_{j=1}^{N} w_j \left[ \tilde{I}^m(\tau, \mu_j)\hat{P}_l^m(\mu_j) + \tilde{I}^m(\tau, -\mu_j)\hat{P}_l^m(-\mu_j) \right] \\
+ \hat{S}^m(\tau, \pm \mu_i, \mu_0); \quad i = 1, 2, \cdots, N
\]

Eigensolutions of these equations could be complex, instead of real only as for scalar case.
Solutions of the RTE are of course different. ...

We have been implementing the discrete-ordinate VRT solutions into the combined COART-MODTRAN code.

To simplify the implementation, only the case with unpolarized illumination (i.e., natural sun light) is treated and so the number of equations is reduced by half; only the three elements \((l, q, u)\) are calculated and the circular polarization \((v)\) is neglected for now (phase matrix is reduced from 4x4 to 3x3 and becomes symmetric). Under this circumstance, the eigensolutions are real variables as in the scalar case and so the complex matrix decomposition is not required.

The Rayleigh scattering component is most important but is relatively easier to be implemented, and so it is implemented first (done).

Here I report the progress in this VRT extension and show the initial results.
A sensitivity test: the relative error of scalar RT in the upward nadir radiance as a function of Rayleigh scattering optical depth.

Err = \frac{(Scalar – Vector)}{Vector} \times 100
Comparison of the TOA radiances (normalized) for the standard mid-latitude atmosphere and Lambertian surface (a=0.1, SZA=45).
The relative difference:

$$100 \frac{(SRT-VRT)}{VRT}$$
Relative error in the principal plane.

![Graph showing difference in reflectance with view angle for SZA=45 degrees. The graph includes lines for wavelengths 350 nm, 550 nm, and 850 nm, with backward and forward view angles indicated.](image)
Error from scalar approximation in the azimuthally averaged radiance is much smaller!
SZA=60

350 nm
550 nm
850 nm

Solid line: Principal plane
Dashed line: Azimuthal average

View Angle
Backward \(\leftarrow\) | \(\rightarrow\) Forward

Difference in Reflectance (%)
Comparison of the TOA radiances over ocean surface 
($\lambda=550\text{nm}, \text{wind}=8\text{m/s}, \text{SZA}=40$).
Comparison of TOA reflectance in the principal plane over ocean ($\lambda=550\text{nm}$).
Relative error in TOA radiance in the principal plane

Ocean surface ($\lambda=550$nm)

For the same 550 nm, the relative error over ocean surface is much larger!
Angular distribution of downward radiance (normalized) in ocean

Wind = 3 (m/s)

Wind = 9 (m/s)

Wind = 18 (m/s)

ARF at 200m in Ocean

ARF at 100m in Ocean

ARF at 10m in Ocean

ARF at 0m in Ocean

ARF at TOA

ARF at TOA - Broadband

ARF at TOA - 865 nm

ARF at TOA - 531 nm
Reflectance at The Angle With Largest Difference (Sun-glint)
Largest Difference (Scalar - Vector)

$\text{Err} = \frac{(\text{Scalar} - \text{Vector})}{\text{Vector}} \times 100$

Reflectance Difference (%)

View angle with largest positive error

View angle with largest negative error

Ocean

Wavelength (nm)
Error in Azimuthally Mean Radiance

![Graph showing reflectance difference against wavelength for different zenith angles: 30 deg (black), 40 deg (red), and 60 deg (green). The graph plots reflectance difference (%) against wavelength (nm) with a Correlated Double Difference (CDD) retrieval and an ocean model.]
Polarization effect on flux or albedo is negligible!
Ocean
Reflectance at The Angle With Largest Difference

The case over desert surface.
Err = (Scalar-Vector)/Vector x 100

View angle with largest positive error

View angle with largest negative error

Desert
Error in Azimuthally Mean Radiance

Zenith angle:
- 10 deg
- 30 deg
- 60 deg
- 80 deg
TOA Albedo

Scalar (solid line)
Vector (dash line)

Wavelength (nm)

Albedo Difference (Scalar-Vector)

Desert
**Summary**

1) The vector RT solution has been implementing into the COART-MODTRAN code. The inclusion of Rayleigh scattering, the largest polarization error source, has been completed.

2) The preliminary results show that the SRT error in the TOA radiance/reflectance varies greatly with molecular optical depth, wavelength and view angle, but this error is mostly less than 10%.

3) The largest SRT error occurs at optical depth around 1.0, corresponding to wavelength about 330 nm for a standard atmosphere, and in or near the principal plane. The error in the azimuthally averaged TOA radiance is significantly smaller and is minimum in the irradiance or albedo.

4) The VRT calculation is a order of magnitude slower than the SRT. With the new polarization implementation, we can now determine when the VRT is required for the spectral simulation, based on the accuracy requirement.
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5) The VRT implementation has been simplified and the computation efficiency is not a consideration for now. More works are required ...